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Thus

$$\tau = \frac{\frac{\pi}{8} C_{\rm D} \frac{1}{2} \rho u^2}{1 + B \left( \frac{\frac{1}{2} \rho u^2}{p_{\rm s} - p_{\rm v}} \right)}$$
$$= \frac{\mu_{\rm w} (p_{\rm s} - p_{\rm v})}{1 + \left( \frac{p_{\rm s} - p_{\rm v}}{\frac{B}{2} \rho u^2} \right)}, \qquad (12)$$

where  $\mu_w$  is a dimensionless coefficient involving B and C<sub>D</sub>:

$$\mu_{\rm w} = \frac{\pi C_{\rm D}}{8B} \quad . \tag{13}$$

The model equation (12) is consistent, of course, with the more general expression (11).

The constant B is approximately 0.5 according to [8], so the bracketed quantity in the denominator of (12) is about 4.0 d/R, generally rather small compared with unity, and very much smaller than unity if the inequalities (5) are taken seriously. A remarkably simple friction law follows,

$$\tau = \mu_{\rm w}(p_{\rm s} - p_{\rm v}) \tag{14}$$

within the domain of validity for this theory. The water exerts <u>Coulomb friction</u> upon the rock, a friction proportional to normal pressure but independent of flow speed. The absence of u in (14) may seem paradoxical, since the surface stress  $\tau$  is due entirely to ram impact of water against the grains. Clearly  $\tau$  should equal zero when u does. The original friction law (12) involves u and implies that  $\tau$  and u go to zero simultaneously. Under conditions of strong cavitation, however, an increase in u decreases the contact between water and grains, and the decreased contact exactly compensates the increased dynamic pressure.

Equation (13) permits a rough estimate of  $\mu_w$ . The constant B is about 0.5 as mentioned before, and  $C_D$  is about 0.4 for cavitational flow around a sphere and 0.8 for a flat disc [9]. The drag of an irregular grain should lie somewhere between that of a sphere and a disc, so the coefficient of Coulomb friction is